

Two dimensional plane jet with the Bickley-Schlichting velocity profile [1]

$$U_0 = 1 - \text{th}^2 \xi$$

( $\xi = y/L$ ,  $U_0 = \bar{U}/U_{00}$  are the nondimensional coordinate and velocity,  $L = 3.64 x^{2/3} (K/\nu^2)^{-1/3}$ ,  $U_{00} = 0.4543 (K^2/\nu x)^{1/3}$  are the reference length and velocity,  $K = \int_{-\infty}^{\infty} \bar{U}^2 dy \equiv \text{const}$ ) is an in-

teresting problem in stability theory since it belongs to the class of flows with a point of inflection in the velocity profile and according to Rayleigh's theorem it has inviscid instability. The stability of plane jet with respect to infinitely small two-dimensional disturbances has been studied theoretically in great depth so far for the plane-parallel flow approximation [2-5] as well as for the nonparallel mean flow [4-6]. Consideration of finite amplitude disturbances [7] makes it possible to theoretically describe the experimentally observed zone of stable oscillations in the neighborhood of critical Reynolds number  $R_*$ . This leads to an increase in  $R_*$  so that the computed and experimental results get closer to each other. There are no studies on the effect of three-dimensional disturbances on the stability of plane jet though, as mentioned in [8, 9], in view of the fact that the jet is very sensitive to external factors like noise, slightest vibrations, "draft of air," it is possible that three-dimensional disturbances could be present in the disturbance spectrum. The appearance of three-dimensional waves can also be the result of the growth of plane disturbances in the nonlinear region. An interesting feature of the spatial wave motion is the creation of longitudinal vorticity in the flow which leads to additional redistribution of momentum in the boundary layer. As shown in [10], one of the simplest models describing the appearance of nonzero moment is the model of cross disturbances, viz., a pair of oblique intersecting Tollmien-Schlichting waves. For a boundary layer on a solid surface such symmetric crossing can form streamwise, spatially periodic vortices as observed in experiments. Considering the fact that the boundary layer on a flat plate is the limiting case of the class of flows with inflection point ( $U_0 \xi \xi = 0$  lies at the wall) and the need for taking three-dimensional disturbances into account has been established, it appears important to study the interaction of such disturbances with inviscidly unstable boundary layer, viz., plane jet, which could be useful to clarify the question of simultaneous effect of disturbance waves of different amplitudes on the stability of such a boundary layer. These studies have been carried out for a plane-parallel jet within the framework of single harmonic approximation [11] which makes it possible to include the system of Reynolds equations for the mean flow. The interaction of oblique disturbances in the form of intersecting oblique Tollmien-Schlichting waves of finite amplitude with the plane flow without solid boundaries, viz., laminar jet with Bickley-Schlichting velocity profile has been considered in this paper. On the basis of an accurate solution of Reynolds equations longitudinal vortices and secondary mean flow have been found and it has been shown that the momentum redistribution in such an interaction from the jet axis to the outer region agrees with the experimental results. The secondary flow happens to be more stable than the basic flow. It should be recognized that the interaction of plane waves and the flow is the critical factor for excitation in the transition region.

1. Symmetric crossing of waves occurs due to the interaction of two oblique Tollmien-Schlichting waves  $\kappa \{u, v, w, p\}_{1,2}(\xi) \exp [i\alpha(x - Ct) \pm i\gamma z]$  with given strength  $\kappa$ , which is present here as a parameter. The angle of inclination  $\theta$  of the waves to the longitudinal axis is determined by the relation  $\tan \theta = \gamma/\alpha$ , where  $\alpha$  and  $\gamma$  are the wave numbers in the streamwise ( $x$ ) and spatial ( $z$ ) directions. If  $\gamma$  is real, then the crossing gives a solution of the type of standing waves whose period is determined solely by the value of  $\gamma$ . The resultant waves are represented as follows

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Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 6, pp. 69-76, November-December, 1982. Original article submitted June 26, 1981.

$$\begin{aligned}
u'_1 + u'_2 &= 2\kappa u(\xi) \sigma \cos \gamma z, & v'_1 + v'_2 &= 2\kappa v(\xi) \sigma \cos \gamma z, \\
w'_1 + w'_2 &= 2\kappa i w(\xi) \sigma \sin \gamma z, \\
p'_1 + p'_2 &= 2\kappa p(\xi) \sigma \cos \gamma z, & \sigma &= i\alpha(x - Ct).
\end{aligned}$$

Using the equations of motion it is possible to show that the following equations are true for disturbance amplitude

$$\{u, v, w, p\}(\xi) = \{u_1, v_1, w_1, p_1\}(\xi) = \{u_2, v_2, -w_2, p_2\}(\xi).$$

Using Squire's transformation for the new variables

$$\begin{aligned}
k^2 &= \alpha^2 + \gamma^2, & p_0 R &= p \text{Re}, & \chi &= \alpha u - \gamma w, \\
kR &= \alpha \text{Re}, & \kappa u_0 &= \alpha u + \gamma w, & \text{Re} &= U_{00} L / \nu
\end{aligned} \tag{1.1}$$

it is possible to get from the linearized Navier-Stokes equations for three-dimensional disturbances a system of equations equivalent to Orr-Sommerfeld equations for two-dimensional disturbances

$$u_{0\xi\xi\xi} - Au_0 = RU_{0\xi}v + ikp_0, \quad v_{\xi\xi} - Av = Rp_{0\xi}, \quad v_{\xi} + iku_0 = 0,$$

along with a nonhomogeneous equation for  $\chi$ :

$$\chi_{\xi\xi} - A\chi = \gamma \text{Re} U_{0\xi} v, \quad A = k^2 + ikR(U_0 - C).$$

The symmetry of the velocity profile with respect to the similarity variable  $\xi$  (as in the case of channel flow), splitting the disturbances into symmetric and antisymmetric parts is generally made to simplify computations. For the antisymmetric mode  $u_0 = v\xi\xi\xi = 0$  along the jet axis ( $\xi = 0$ ) and for the symmetric mode  $u_{0\xi} = v = 0$ . At the outer boundary of the jet  $\xi = \xi_k$  (usually  $\xi_k = 6$ ) the condition of the boundedness of disturbances is stipulated [3, 5]:

$$\begin{aligned}
u_{0\xi\xi\xi} + (k + \beta)u_{0\xi} + k\beta u_0 &= 0, \\
v_{\xi\xi} + (k + \beta)v_{\xi} + k\beta v &= 0, \quad \beta^2 = A(\xi_k).
\end{aligned}$$

The behavior of the even component of velocity disturbances  $w$  coincides with the even component  $u$  and hence for the antisymmetric mode it follows from (1.1) that  $\chi(0) = 0$ , and for the symmetric mode  $\chi_{\xi}(0) = 0$ . It is possible to put, as a first approximation,  $\chi(\xi_k) = 0$  for both modes at the jet boundary.

As shown by computations and experiments the jet instability is determined by antisymmetric disturbances for which it has been found that  $R_* \sim 4-8$ , and for symmetric  $R_* \sim 88$  (3, 5). Consequently, the primary attention in this paper is paid to the antisymmetric mode as the most unstable.

2. The secondary flow in the quasisteady approximation is described by Reynolds system [1] which is written in the following form after nondimensionalizing

$$VU_{\xi} + WU_z = (1/\text{Re})(U_{\xi\xi} + U_{zz}) - f_1 - f_0, \tag{2.1a}$$

$$VV_{\xi} + WV_z = -P_{\xi} + (1/\text{Re})(V_{\xi\xi} + V_{zz}) - f_2; \tag{2.1b}$$

$$VW_{\xi} + WW_z = -P_z + (1/\text{Re})(W_{\xi\xi} + W_{zz}) - f_3,$$

$$V_{\xi} + W_z = 0.$$

Here  $\xi$  is taken as the transverse similarity coordinate. Reynolds stresses  $f_1 - f_3$  are obtained by statistical averaging of corresponding second order moments while for the given system of waves they do not depend on the coordinate  $x$ ;  $f_0$  takes the form of external pressure. The resulting secondary flow will have the form  $U = U(\xi, z)$ ,  $V = V(\xi, z)$ ,  $W = W(\xi, z)$ , and it is natural to choose the following boundary conditions: the condition of boundedness of flow along the coordinate  $\xi$ :  $\{U, V, W\}(\xi, z) \rightarrow 0$  as  $\xi \rightarrow \pm\xi_k$  and the periodicity condition along the coordinate  $z$ :

$$\{U, V, W, U_z, V_z, W_z\}(\xi, 0) = \{U, V, W, U_z, V_z, W_z\}(\xi, T).$$

It is clear that in such a formulation it is possible to find  $V, W$  independent of  $U$ . The secondary flow  $V, W$  in the "stream function-vorticity" variables

$$V = \Psi_z, \quad W = -\Psi_{\xi}, \quad \omega = V_z - W_{\xi}$$

is determined from the following system:

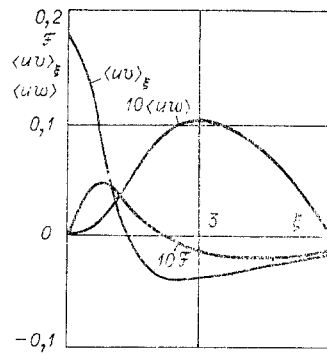


Fig. 1

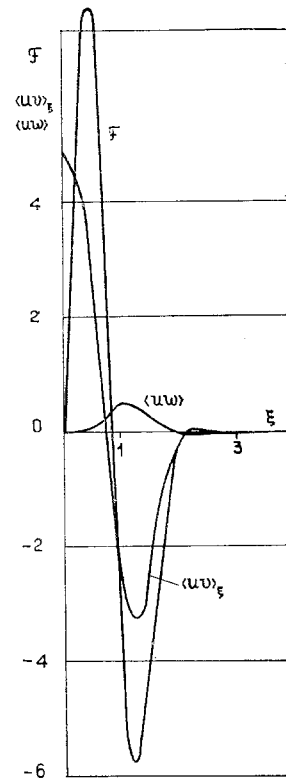


Fig. 2

$$\begin{aligned} \Psi_z \omega_\xi - \Psi_\xi \omega_z &= (1/\text{Re})(\omega_{\xi\xi} + \omega_{zz}) - F(\xi, z), \\ \omega &= \Psi_{zz} + \Psi_{\xi\xi}, \quad F(\xi, z) = f_{2z} - f_{3\xi}. \end{aligned} \quad (2.2)$$

For the crossing  $F(\xi, z) = -\kappa^2 \mathcal{F}(\xi) \sin 2\gamma z$ , where  $\mathcal{F}(\xi) = \langle vw \rangle_{\xi\xi} + 2\gamma[\langle vw \rangle_\xi + \langle w \omega \rangle_\xi + 2\gamma \langle vw \rangle]$ . It follows from the form of  $F$  that the period  $T$  is determined as  $T = 2\pi/2\gamma$ . For the Blasius boundary layer, the linearized approximation (2.2) which looks exactly like the equation for the secondary flow within the framework of the method of small disturbances, makes it possible to obtain a sufficiently accurate solution to the complete problem for moderate values of  $\kappa$ , when convection plays a secondary role, and the extremal solution for large  $\kappa$  [10]. For the first term of the Fourier series  $\Psi(\xi, z) = \psi(\xi) \sin 2\gamma z$ , the linearized system (2.2) can be brought to the following nonhomogeneous fourth order ordinary differential equation:

$$\psi_{\xi\xi\xi\xi} - 8\gamma^2 \psi_{\xi\xi} + 16\gamma^4 \psi = -\text{Re} \kappa^2 \mathcal{F}. \quad (2.3)$$

Its boundary conditions are  $\psi, \psi_\xi \rightarrow 0, \xi \rightarrow \pm \xi_k$ . It is possible to conclude from physical considerations that for antisymmetric disturbances  $\psi = \psi_{\xi\xi} = 0$  on the jet axis, and for symmetric disturbances  $\psi_\xi = \psi_{\xi\xi\xi} = 0$  which again makes it possible to limit the domain of the solution. At the outer boundary  $\xi = \xi_k$  the exponential decrease of  $\psi$  in the form  $\psi \sim (D_1 + D_2 \xi) \exp[-2\gamma \xi]$  leads to the following relations:

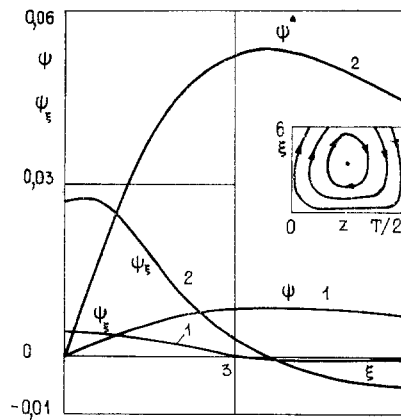


Fig. 3

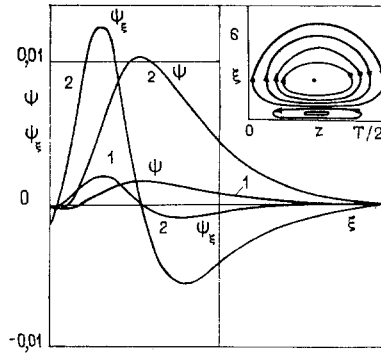


Fig. 4

$$\psi_{\xi\xi} + 4\gamma(\psi_{\xi} + \gamma\psi) = 0, \quad \psi_{\xi\xi\xi} + 4\gamma(\psi_{\xi\xi} + \gamma\psi_{\xi}) = 0,$$

which can be taken as the boundary conditions for both modes. Knowing the solution (2.3), we determine  $V(\xi, z) = 2\gamma\psi(\xi)\cos 2z$ ,  $W(\xi, z) = -\psi\xi \sin 2\gamma z$ . The longitudinal component of the secondary flow  $U(\xi, z)$  is described by the Eq. (2.1a) which is solved with the help of implicit difference scheme of second order accuracy by iteration [13]. The choice of external pressure  $f_0$  is determined by the condition that as  $\kappa=0$ , and in the absence of secondary  $V$  and  $W$ , Eq. (2.1a) gave a laminar profile for the plane jet  $U_0$ . Hence it is assumed that  $f_0 = U_0\xi\xi/\text{Re}$  which agrees with [7]. The domain of the solution (2.1a) is the rectangle  $[-\xi_k \leq \xi \leq \xi_k, 0 \leq z \leq T]$ , the boundary conditions are given above. In view of the fact that the boundary  $\xi_k$  is chosen sufficiently far (in the laminar case  $U_0(\pm\xi_k, z) \sim 10^{-5}$ ), along with the boundary condition  $U(\pm\xi_k, z) = 0$  based on physical considerations, the condition  $U(\pm\xi_k, z) = 0$  is also considered that gave very well-coinciding distributions  $U(\xi, z)$ . Reynolds stresses  $f_1$  for the intersections can be given the form  $f_1 = \kappa^2[\langle uw \rangle_{\xi} + 2\gamma\langle uv \rangle] \cos 2\gamma z + \langle uv \rangle_{\xi}$ . It has a periodic part which can exist only for three dimensional fluctuations and the second part which is analogous in form to Reynolds stresses for two dimensional disturbances [7] through which the redistribution of momentum is carried out for the plane flow in the field of plane wave disturbance. Analysis of the simultaneous as well as the independent effect of the components of  $f_1$  on the mean flow can give the answer to the question of concurrent effect of two- and three-dimensional disturbances on the flow.

3. For antisymmetric disturbances two domains of three-dimensionality have been studied. These characterize small ( $\theta \sim 6.5^\circ$ ) and large ( $\theta \sim 35^\circ$ ) angles of inclination of the wave to the longitudinal axis. Table 1 gives the eigenvalues of three-dimensional ( $\alpha, \gamma\text{Re}, C$ ) and equivalent two-dimensional ( $k, R, C$ ) waves for four points with the upper branch of the neutral curve [5] and indicates the range of the angles  $\theta$  that have been studied for the critical Reynolds number of symmetric disturbances (point 5).

Figures 1 and 2 show the Reynolds stress distribution  $\langle uv \rangle_{\xi}, \langle uv \rangle, \mathcal{F}(\xi)$  for the points 1 (Fig. 1) and 3 (Fig. 2). At low  $R$  (points 1, 2) the moment  $\langle uv \rangle_{\xi}$  dominates in the jet and it is responsible for the plane distortion of the flow. With an increase in Reynolds number, as observed from a comparison, there is an increase in the moment  $\langle uv \rangle$  and the complex  $\mathcal{F}$ ,

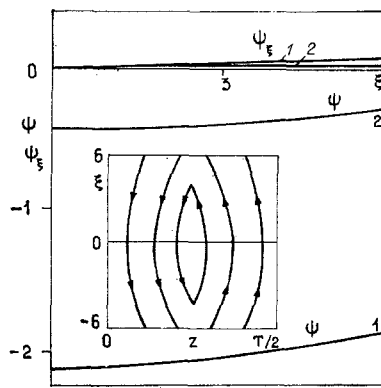


Fig. 5

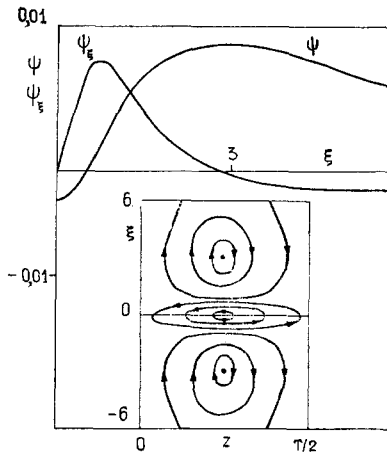


Fig. 6

which determine the spatial distortion of the flow and the strength of the induced vortex. The secondary vortex is shown in subsequent Figs. 3-7 in the form of amplitude functions  $\psi(\xi)$  and  $\psi_\xi(\xi)$  for  $\kappa = 0.02$  and  $0.05$  (lines 1, 2 respectively). These values of  $\kappa$  for the chosen normalized eigenfunctions indicate that the amplitude of disturbances is  $\sim 2$  and  $5\%$  respectively of the mean flow velocity. The streamlines  $\Psi = \text{const.}$  are schematically shown here. For antisymmetric disturbances the vortex center is located on the line  $z = T/4$  and the complete picture of the disturbance to the flow region is given by symmetrical vortices with respect to  $\xi = 0$  and  $z = T/2$ . At small angles  $\theta$  for all  $R$ . The vortex picture is similar to the one shown in Fig. 3 ( $R = 19.45$ ,  $\theta = 6.5^\circ$ ). For small  $R$  streamlines are not closed, the assumed center lies outside the flow domain in the stagnation region, and a localization of the vortex in the flow region takes place with increase in  $R$ . For the points 1, 2 the structure is similar to the one drawn for large  $\theta$  but starting from  $R = 19.45$  the picture for large  $\theta$  is enriched by weak axial counter-rotating vortices shown in Fig. 4 for the point 3 when  $\theta = 34.8^\circ$ . We observe that the strength of induced vortices is inversely proportional to  $\theta$ .

An interesting vortex structure has been obtained by disturbing the flow with crossed symmetric waves. Computations showed that at small angles  $\theta$  one vortex is induced in the jet in the entire flow region  $-\xi_k \leq \xi \leq \xi_k$ , and the complete picture consists of two symmetric vortices with respect to  $T/2$  (Fig. 5 for  $\theta = 6.6$  and  $13.35^\circ$ ; lines 1, 2 respectively). With an increase in  $\theta$  the structure becomes more complex: There is a degeneration of the initial vortex (reduction in strength and mixing in the neighborhood of the axis) and the development of two new counter-rotating vortices at large (Fig. 6 for  $\theta = 27.5^\circ$ ). Further increase (Fig. 7 for  $\theta = 43.85^\circ$ ) leads to the complete disappearance of this near-axis vortex and a merging of the rest.

Such is a fairly complex structure of vorticity developed in the plane jet disturbed by the crossing of Tollmien-Schlichting waves. The streamwise velocity component of the

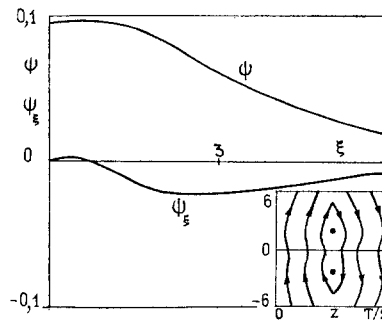


Fig. 7

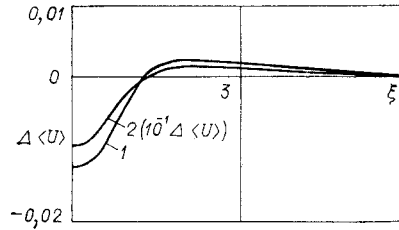


Fig. 8

TABLE 1

Point No.	$\alpha$	$\gamma$	$\theta^\circ$	Re	$k$	R	$C=C_r+iC_i$
Antisymmetric Disturbance							
1	0,1688	0,02	6,8	4,04	0,168	4,0155	
	0,135	0,1	35,8	5,00			
2	0,8709	0,1	6,5	10,21	0,8766	10,14	0,36574 -0,00292i
	0,72	0,5	34,8	12,35			
3	1,2205	0,14	6,5	19,58	1,2285	19,45	0,4636 +0,00058i
	1,01	0,7	34,7	23,67			
4	1,5343	0,175	6,43	41,13	1,5443	40,867	0,5546 -0,0001i
	1,2725	0,875	34,8	49,59			
Symmetric Disturbance							
5 $R_*$	0,4301	0,05	6,63	88,89	0,433	88,292	0,71597 +0,0i
	0,4213	0,1	13,35	90,75			
	0,38404	0,2	27,51	99,55			
	0,3122	0,3	43,85	122,44			

secondary flow  $U(\xi, z)$  obtained from (2.1a) is represented in the form  $\langle U(\xi) \rangle = \frac{1}{T} \int_0^T U(\xi, \zeta) d\zeta$

which is convenient for comparison with the laminar profile. Integration is carried out using the approximate Simpson's formula. Results on the flow distortion are given in the form of velocity defect  $\Delta\langle U(\xi) \rangle = \langle U(\xi) \rangle - U_0$ . The redistribution of momentum takes place from the neighborhood of the axis (small  $\xi$ ) to the outer region of the flow field (large  $\xi$ ). The velocity defect is the maximum in value at  $\xi = 0$  where  $U_0 = 1$ . For large  $\xi$ , where  $U_0 \sim 10^{-2}$  to  $10^{-3}$ , the distortion can approach 50% of the laminar case but in view of small  $U$  it is not clear whether it is caused by the physical process and not by computational errors (in particular, boundary condition at  $\xi_k$ ). It was observed that the distortion increases with increase in Reynolds number, angles  $\theta$  and strengths  $\kappa$ . A typical distortion of the mean velocity profile (velocity defect) is shown in Fig. 8 for the point 3 ( $\theta \sim 35^\circ$ )  $\kappa = 0.02$  and  $0.05$  (lines 1 and 2 respectively). Qualitatively this picture is similar to this even for other  $R$ . By summing up the results for a monotonically increasing series of

TABLE 2

		R=10,14		R=19,45		R=40,866	
$\theta$		6,5°	34,8°	6,5°	34,7°	6,43°	34,8°
$\kappa$							
0,02		0,363213 -0,0069i	0,355091 -0,0069i	0,464052 +0,0004i	0,46555 -0,002i	0,55566 -0,0013i	0,56248 -0,0034i
0,05		0,34747 -0,004i	0,35287 -0,0043i	0,47199 -0,0027i	0,48372 -0,0074i		

Reynolds numbers we get the picture for the qualitative variation of the mean flow which corresponds to the experimentally observed structure. It is characterized by the flattening of the velocity profile  $\langle U(\xi) \rangle$  with increasing R. The degree of "pulsation" of such a flow is  $\varepsilon_{\Pi} = [\langle (w')^2 \rangle + \langle (v' + V)^2 \rangle + \langle (w' + W)^2 \rangle] / 3l^{1/2}$ , which essentially is the same as the degree of turbulence  $\varepsilon_T$ , is approximately equal to the chosen strength of the disturbances  $\kappa$ .

The stability of the mean flow  $\langle U(\xi) \rangle$  has been carried out within the framework of linear theory. Results are given in Table 2, the initial eigenvalues  $C = C_R + iC_i$  are indicated in Table 1. It was observed that as a result of distortion the flow becomes more stable when compared to the basic flow which agrees with the data [7] on the stability of self-sustained oscillation regimes and experimental data. Equations (2.1a) for the flow averaged with the assumption that the streamwise vortex is absent and the only term retained in Reynolds stresses  $f_1$  being  $\kappa^2 \langle uv \rangle_{\xi}$ , which determines the plane interaction, is reduced to the form analyzed in [7]. It was observed that for small values of Reynolds number (points 1 and 2) the distortion of the mean flow is determined by the purely plane interaction, the vortex strength is so small that it practically does not contribute anything to the energy redistribution. With an increase in R the influence of three dimensional disturbance is felt which strengthens the effect of plane wave. For the point 3 this effect is estimated to be approximately 1-2% and for the point 4 it already approaches 10%. The nature of the disturbance waves should indeed be felt from these Reynolds numbers.

The results obtained show that for a boundary layer without solid boundaries (plane Bickley-Schlichting jet) which is inviscid unstable within the framework of linear stability theory, three-dimensionality of disturbances could play a role only in the range of sufficiently large Reynolds numbers ( $R \sim 40$ ); however for Reynolds numbers lying close to the experimentally observed transition R, the main feature of the disturbed flow is described by plane interaction which fundamentally differentiates this flow from the boundary layer on a solid surface.

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INVESTIGATION OF JET FLOW PAST SLOTTED AND WEDGE-SHAPED NOZZLES  
IN A SHOCK TUBE

V. V. Golub, V. V. Grigor'ev,  
Yu. I. Grin', S. N. Isakov,  
I. M. Naboko, R. L. Petrov,  
and V. G. Testov

UDC 533.601.1:534.202.2

Studies in shock tubes have been extensive in recent years. These studies are directed in search of techniques to increase the effectiveness of gasdynamic lasers in which, as a rule, plane sonic and supersonic nozzles are used for the production of a jet issuing into free space or channel. The published experimental studies are primarily devoted to the measurement of quantum characteristics of GDL (amplification factor, power developed). At the same time, gasdynamic studies are few and concern mainly the determination of the wave structure of the jet, though relaxation of vibrational energy is determined by the distribution of gasdynamic parameters in the flow: velocity, temperature, pressure, and density. In computing the properties of gasdynamic lasers it is usually assumed that the jet is one-dimensional and steady. However, experimental studies and computations [1-5] of jets brought out a number of significant features of the wave structure and the distribution of jet parameters. It was shown that the flow past a nozzle section can have a fairly complex spatial structure which affects the characteristics of the laser beam. In particular, flow nonuniformity leads to phase nonuniformities in the laser beam which has an important bearing on the operation of laser at increased power conditions. Besides, in experiments with nozzles in shock tube it is necessary to keep in view that a transient flow process precedes quasisteady jet efflux. In the present paper results are given for the experimental studies on three dimensional and plane jets in shock tubes under conditions similar to those in which studies on the laser characteristics [7, 8] of gas flows were conducted: transient time for the density field and the flow geometry, spatial characteristics of density distribution.

Measurements were made in shock tubes with low pressure channel of cross-section  $40 \times 40$  and  $35 \times 70$  mm. Plane sonic nozzles were set up at the low pressure end in the form of orifices with cross section  $h \times a$  equal to  $1.5 \times 40$  and  $2.5 \times 70$  mm ( $a/h = 27$  and  $28.5$ ) or plane wedge-shaped supersonic nozzle with an aperture angle of  $30^\circ$  and area ratio  $A_\alpha/A^* = 15$  at a height  $h = 1.3$  mm.

During studies on spatial jet with sonic nozzle in the shock tube with square cross-section, the low pressure chamber and the reservoir were filled with nitrogen. The initial pressure  $p_1$  was 36 GPa, Mach number  $M_1$  of the incident shock wave varied in the interval  $M_1 = 2.5-3.5$ , and the degree of expansion  $n = p_\alpha/p_\infty = 16-42$ . In the case of wedge-shaped nozzle the degree of expansion was varied in the range 15-70.

Measurements in rectangular shock tube for the sonic nozzle with  $a/h = 28.5$  were made without the nozzle diaphragm with the initial pressure in the low pressure chamber and the decompression chamber  $p_1 = 133$  GPa and degrees of expansion  $n = 7.9$  ( $M_1 = 2.0$ ) and  $n = 12.8$  ( $M_1 = 2.2$ ). When the Mach number of the incident shock wave  $M_1 = 1.9$ ,  $p_\infty = 1.33; 13.3; 26.6; 53.2$  GPa and  $p_1 = 0.1$  MPa, the following parameters were obtained: ahead of the nozzle  $p_s = 1.9$  MPa,  $\rho_s = 6.43$  kg/M<sup>3</sup>, and at the nozzle cross section  $M_\alpha = 4.35$ ,  $p_\alpha = 53.7$  GPa,  $\rho_\alpha = 0.1286$  kg/M<sup>3</sup>. The measured time for the existence of constant deceleration parameters before the nozzle was  $\sim 2.5$  msec.

The flow in the  $40 \times 40$  mm shock tube was visualized using the shadowgraph IAB-451. The investigation of the unsteady structure of three dimensional jet was conducted with the

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Moscow. Translated from *Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki*, No. 6, pp. 76-80, November-December, 1982. Original article submitted October 28, 1981.